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Tensile strength of fibre-reinforced metal matrix composites with non-uniform fibre spacing

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The influence of non-uniform fibre spacing on the strength of unidirectional fibre-reinforced metal matrix composites was studied by means of a Monte-Carlo computer simulation experiment. The influence of yield stress of the matrix and scatter of the fibre strength on the strength of composites were also studied for both uniform and non-uniform fibre spacings. It was demonstrated that (1) the strength of composites with non-uniform fibre spacing is lower than that with uniform spacing due to the high stress concentration arising from the breakage of fibres, and (2) the reduction in strength of composites due to the non-uniformity increases with increasing scatter of fibre strength. For both cases of uniform and non-uniform spacings, the following tendencies were observed : (a) the strength of composites increases but then decreases with increasing yield stress of matrix, (b) it is very sensitive to yield stress of the matrix when the scatter of fibre strength is large but not when it is small, and (c) it decreases but then increases with increasing scatter of fibre strength when the yield stress of the matrix is high, while it decreases monotonically with increasing scatter of fibre strength when the yield stress is low.

1. Introduction

When fibres are supplied in a bundle form, it is difficult to separate the fibres well from each other and also to fabricate composites with uniform fibre spacing. Thus, in practical composites, the fibre spacing is, more or less, non-uniform. When fibre spacing is non-uniform, a high stress concentration is exerted on the fibres adjacent to broken fibres in comparison with the stress concentration for uniform fibre spacing, as shown in our preceding paper [1]. It is speculated that this high stress concentration for non-uniform fibre spacing will act to reduce the strength of composites from the strength for uniform array. This speculation has been verified by Towata and Yamada [2] who have found that the strength of SiC (made from polycarbosilane)-reinforced aluminium can be improved by separating the individual fibres well from each other. However, except for these data, there are no other available data on the influence of non-uniformity of fibre spacing on the strength of fibre-reinforced metals, although it has been requested that the influence be studied in detail. At present, it is unknown how the fibre spacing affects the strength under a wide range of hardness of matrix, interfacial bonding strength, scatter of strength of fibres, and so on.

The aim of the present work was to study the influence of non-uniformity of fibre spacing on the strength of continuous fibre-reinforced metal matrix composites whose interfacial bonding strength is high enough to suppress debonding at the interface under wide range of matrix hardness and scatter of strength of fibres. In this work, a computer simulation technique [3–6] based on the Monte-Carlo method was employed because various factors can be changed at will, which will provide wide information on this problem.

2. Experimental procedure

Computer simulation experiments were carried out using two-dimensional model composites for the two cases: Case A where fibre spacing is uniform as shown in Fig. 1a and Case B where it is non-uniform as shown in Fig. 1b. In both cases, the composite was regarded as being composed of a number of M_2 fibres, each fibre being composed of a number of M_1 elements with a length δ in longitudinal direction. For Case A, the procedure of the simulation experiments was the same as that employed in our previous work [3, 4, 6]. For Case B, the model composite was modified as follows: every three fibres ((1, 2, 3), (4, 5, 6), (7, 8, 9), . . .) makes a group, and the group with wide fibre spacing ((1, 2, 3), (7, 8, 9), . . .) and that with narrow spacing ((4, 5, 6), (10, 11, 12), . . .) exist one after another, as shown in Fig. 1b, where the wide fibre spacing is noted as d_A and the narrow one as d_B. In this work, the ratio of d_A/d_B was taken to be 10. The procedure of the simulation experiments was the same as that for Case A with the exception stated above.

The input values are listed in Table I. Among the values, shear yield stress of matrix, τ_Y, was varied from 6 to 300 MPa and the coefficient of variation of strength of fibre, CV, was varied from zero to 0.28 where the strength of the fibres was assumed to obey the Weibull distribution function [7]. The simulation

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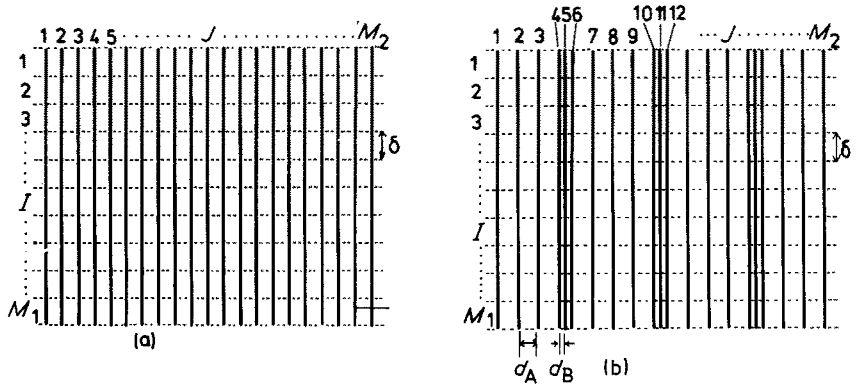

Figure 1 Schematic representation of the model composite: (a) Case A and (b) Case B.

experiments were carried out 25 times or 100 times when necessary for a given condition, and average values, standard deviations and histograms were obtained.

3. Results

Figs 2 and 3 show the variations of average strength and its standard deviation of composites as a function of τ_Y for CV = 0.04 and 0.12, and for CV = 0.23, respectively. Fig. 4 shows the variation of average strength and its standard deviation as a function of CV for the given values of τ_Y = 150 and 6 MPa. Figs 5 and 6 show a comparison of histograms of the strength for Case B with those for Case A for CV = 0.04 and 0.23 under fixed values of τ_Y = 6 and 150 MPa, respectively. The following features could be seen from Figs 2 to 6.

(i) The average strength for Case B is lower than that for Case A for any τ_Y and CV. Namely, the average strength is reduced due to the non-uniformity of fibre spacing.

(ii) The scatter in the strength for Case B is always larger than that for Case A.

(iii) The average strength increases, reaching a maximum, and then decreases with increasing τ_Y for both cases A and B.

(iv) The larger the CV, the larger becomes the difference in strength between Cases A and B.

(v) In both cases A and B, when CV is large, the strength is very low for low τ_Y but very high for optimum τ_Y ($\approx$ 150 MPa). On the other hand, when the CV is small, the dependency of the strength on τ_Y is small. Namely, the strength becomes very sensitive to yield stress of the matrix when CV is large but not when CV is small.

TABLE I Input values employed for the present computer simulation experiments

Number of elements in each fibre, M_1	25
Number of fibres, M_2	27
Length of composite, L	50 mm
Young's modulus of fibre, E_f	400 GPa
Shear modulus of matrix, G_m	40 GPa
Strain hardening coefficient of matrix normalized with respect to G_m, β	0.01
Diameter of fibre, d_f	100 μm
Volume fraction of fibre, V_f	0.5
Average strength of fibre, $\bar{\sigma}_f^0$	3 GPa
Shear yield stress of matrix, τ_Y	6–300 MPa
Coefficient of variation of strength of fibre, CV	0–0.28

(vi) When τ_y is high, the strength decreases, reaching a minimum, and then increases with increasing CV, but when τ_y is low, it decreases monotonically with increasing CV.

In the present work, the fracture mode of composites for Cases A and B under various combinations of values of CV and τ_y was also studied. The results were very similar to those of the former work [3, 4, 6] for Case A under various combinations of values of CV and interfacial bonding strength. The results are summarized as follows.

(a) The fracture of composites occurs in a non-cumulative mode when CV is small; namely breakages of a few weaker fibres cause fracture of composites as a whole.

(b) When CV is large, the fracture of composites occurs in a cumulative mode; namely breakages of weaker fibres are accumulated in composites and after accumulation of the breakages, composites fracture as a whole.

(c) When τ_y is low, pull-out of fibres occurs and the fracture surface is irregular, but when τ_y is high, pull-

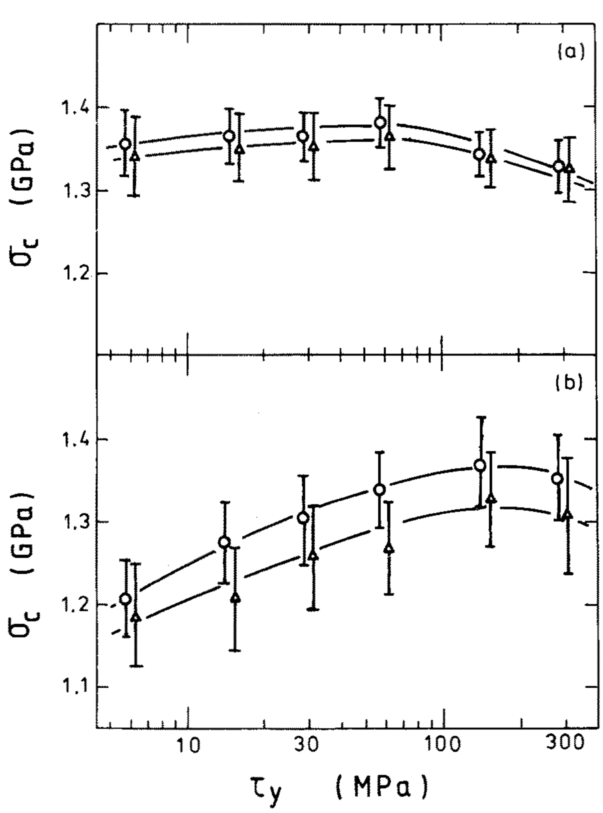

Figure 2 Variation of σ_c as a function of τ_Y for (a) CV = 0.04 and (b) CV = 0.12 for Cases A (O) and B ($\triangle$).

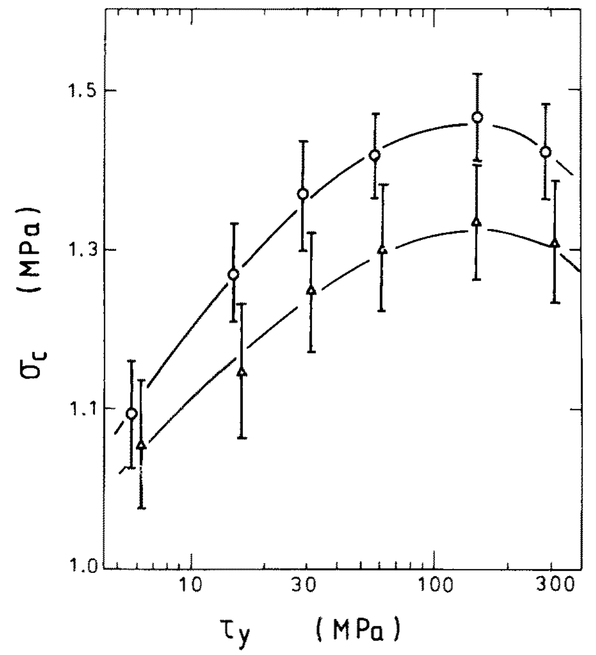

Figure 3 Variation of σ_c as a function of τ_Y for CV = 0.23 for Cases A (○) and B (△).

out of fibres is not allowed due to the short critical length and high stress concentration in the fibres adjacent to the broken fibres, and the fracture of composites tends to occur in one cross-section perpendicular to the tensile axis.

4. Discussion

4.1. Influence of yield stress of the matrix and non-uniformity of fibre spacing on strength of composites

The strength of composites is strongly affected by the stress concentration in the fibres adjacent to broken fibres and also critical length [3, 4, 6]. According to the results of calculations [1, 8] based on the shear-lag analysis, the stress concentration for a given applied stress increases with increasing τ_Y, which acts to reduce the strength of composites with increasing τ_Y for both cases A and B. On the other hand, the critical length decreases with increasing τ_Y, which acts to raise the strength of composites with increasing τ_Y. The strength of composites is thus determined by these contradictory factors of stress concentration and critical length. Within the present study, the strength of the composites increases with increasing τ_Y when

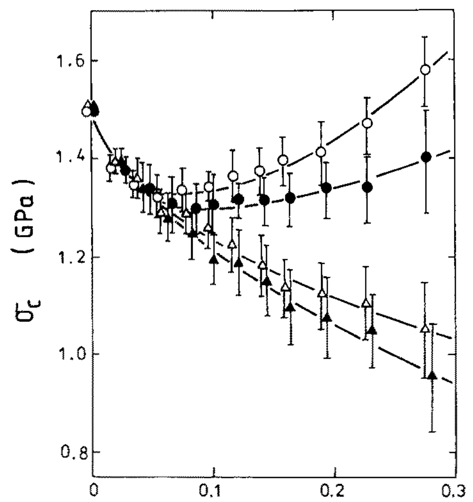

Figure 4 Variation of σ_c as a function of CV for Case A and τ_Y = 150 MPa (○), Case B and τ_Y = 150 MPa (●), Case A and τ_Y = 6 MPa (△), and Case B and τ_Y = 6 MPa (▲).

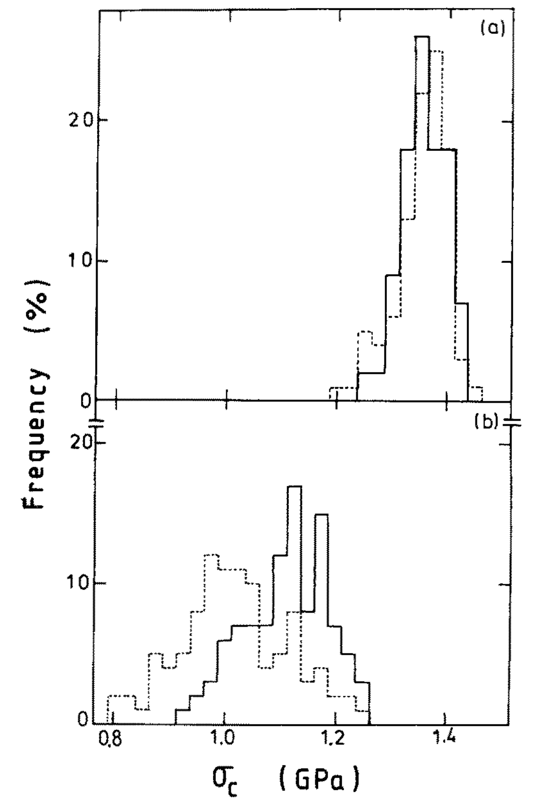

Figure 5 Histogram of the strength values for τ_Y = 6 MPa for (a) CV = 0.04 and (b) CV = 0.23. (——) Case A and (- - -) Case B.

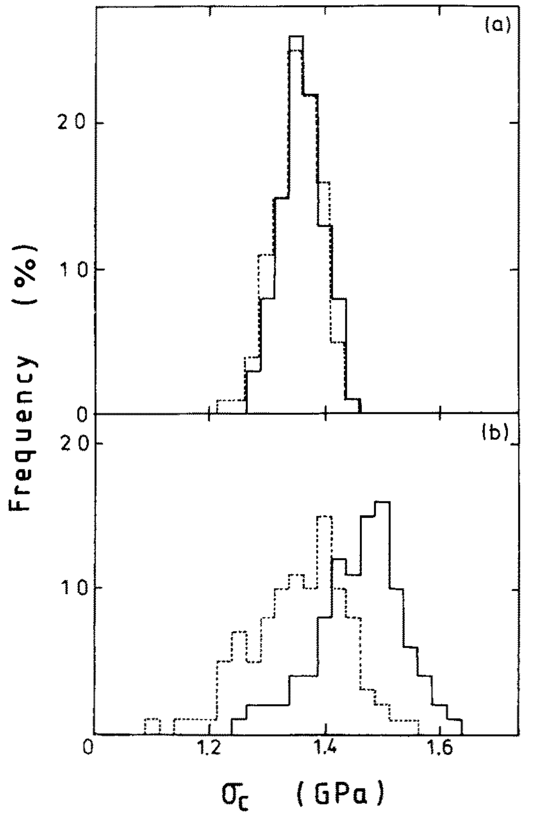

Figure 6 Histogram of the strength values for τ_Y = 150 MPa for (a) CV = 0.04 and (b) CV = 0.23. (——) Case A and (- - -) Case B.

τ_Y is relatively low, indicating that the decrease in critical length with increasing τ_Y can raise the strength of composites, despite the increase in stress concentration. On the other hand, when τ_Y becomes very high, the detrimental effect of increase in stress concentration can reduce the strength of the composites despite the decrease in critical length. Thus the strength of composites increases, reaching maximum, and then decreases with increasing τ_Y for both Cases A and B.

For a given τ_Y, the stress concentration for Case B due to breakage of the fibres in the group with narrow fibre spacing as shown in Fig. 1 is higher than that for Case A, resulting in reduction in strength in Case B in comparison with the strength in Case A. The reduction in strength due to non-uniformity of fibre spacing is dependent not only on τ_Y but also on CV. In Section 4.2, how CV and the non-uniformity affect on strength will be discussed.

4.2. Influence of CV and non-uniformity of fibre spacing on strength of composites

In the present simulation experiments, each fibre was assumed to consist of 25 elements. If one element is broken when τ_Y is low, many elements below and above the broken element within the distance from it to half the critical length become unable to carry the applied stress sufficiently. When the CV is large, many elements are fractured during deformation, leading to a low efficiency of reinforcement. This explains why the strength is low for low τ_Y when CV is large. In contrast, when τ_Y is high, composite fracture tends to occur as a chain reaction of element breakages in one cross-section owing to the short critical length which does not allow fibre pull-out [4, 6]. In order to break the fibres in one cross-section, a high stress is needed for a large CV because the strength of the elements given by the Weibull distribution function [7] is high. For example, the strength of elements for CV values of 0.04, 0.12 and 0.23 are 3.34 ± 0.14, 4.14 ± 0.50 and 5.71 ± 1.31 GPa, respectively, whereas the average strength of fibres with a gauge length is assumed to be 3 GPa for all values of the CV. This explains why the strength becomes high at high τ_Y when the CV is large. In the case of small CV, as the strength of the elements is not very different from each other, fracture of the weakest element tends to cause fracture of the composites as a whole. Thus the yield stress of the matrix has little effect on the strength when the CV is small.

When τ_Y is high, the strength decreases and then increases with increasing CV, but when τ_Y is low, it decreases monotonically as shown in Fig. 4. This is explained as follows. When τ_Y is high, composite fracture tends to occur in one cross-section oriented perpendicular to the tensile axis because fibre pull-out is not allowed, as mentioned above. When the CV is small, breakage of the weakest element results in fracture of the composite as a whole because the strengths of all elements are very similar and therefore the stress concentration due to breakage of the weakest element causes consecutive fracture of neighbouring elements. In this fracture mechanism, as the strength of the weakest element decreases with increasing CV, the strength of the composite also decreases with increasing CV. However, when the CV becomes large the strength of the elements becomes high, as stated above, leading to an increase in strength of composites with increasing CV when τ_Y is high. On the other hand, when τ_Y is low, the efficiency of reinforcement is reduced with increasing CV because large CV yields a large number of breakages of fibres, each of which makes the elements within half the critical length unable to carry applied stress sufficiently. Thus the strength decreases with increasing CV when τ_Y is low.

As stated above, when CV is small, the composite showed a non-cumulative fracture mode for both cases of low and high τ_Y; namely, the composite was broken by the breakage of weaker fibres without accumulation of breakages of fibres. For this case of small CV, the influence of non-uniformity of fibre spacing on the strength can be explained as follows. If a fibre in the group with narrow spacing (for instance, (4, 5, 6) in Fig. 1) is broken, the stress concentration is very high [1], which can break the neighbouring fibres. When a neighbouring fibre is broken, the stress concentration due to two broken fibres becomes higher than that due to one broken fibre, which will cause fracture of the surviving neighbouring fibres. Thus the strength of the composite for Case (B) becomes lower than that for Case A. In contrast, if a fibre in the group with wide fibre spacing (for instance, (7, 8, 9) in Fig. 1) is broken, the stress concentration in the neighbouring fibre is not high [1]. For such a case, the reduction of strength of the composites is minor. This tendency is well realized in the histograms of strength for small CV shown in Figs 5a and 6a. This indicates that the upper bound of the strength for small CV in Case B is nearly the same as that in Case A, but the lower bound in Case B is lower than that in Case A. Thus the average strength is reduced and the scatter of the strength becomes large when the fibre spacing is not uniform. However, as the strength of fibres is not very different from each other when CV is small, the reduction due to the non-uniformity of fibre spacing is small.

In the case of large CV, the average strength is reduced and the scatter of strength becomes large due to the non-uniformity of fibre spacing as well as those in the case of small CV. The reduction in strength is, however, very large. The reason for this can be attributed to the cumulative fracture mode for large CV. When this fracture mode occurs, there arise not only breakages of the fibres in the group with wide fibre spacing but also those in the group with narrow fibre spacing. The latter breakages result in higher stress concentration, resulting in lower strength of composites than the strength for uniform fibre spacing.

5. Conclusions

1. The strength of composites for non-uniform fibre spacing is lower than that for uniform spacing due to the high stress concentration.

2. The difference in strength between the composite whose fibre spacing is uniform and that whose spacing

is not uniform increases with increasing scatter of fibre strength.

3. For both cases of uniform and non-uniform fibre spacings, the following tendencies were observed concerning the influence of yield stress of the matrix and scatter of fibre strength:

(a) the strength of composites increases, reaching a maximum and then decreases with increasing yield stress of the matrix;

(b) the strength of the composites is very sensitive to yield stress of the matrix when scatter of fibre strength is large but not when it is small;

(c) when the yield stress of the matrix is high, the strength decreases, reaching a minimum, and then increases with increasing scatter of fibre strength, but when the yield stress is low, it decreases monotonically with increasing scatter of fibre strength.

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